Total Pages—5

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B.Tech-3
Math-III

Set-2

Full Marks : 70

Time : 3 hours

Q. No. 1 which is compulsory. Answer any **five** questions from the remaining **seven** questions

The figures in the right-hand margin indicate marks

1. Answer *all* parts of this questions : 2 × 10

   (*a*) Show that $u = \tan^{-1}(y/x)$ is a solution for the Laplace equation.

   (*b*) Form a partial differential equation by eliminating the arbitrary function from $z = f(x^2 - y^2)$.

   (*c*) State the physical assumptions of modeling a vibrating string.

   (*d*) Solve $y^2 u_x - x^2 u_y = 0$ by separation of variables.

(*Turn Over*)

(e) Sketch the set $Re(1/z) < 1$ in the complex plane.

(f) If $f(z) = \cos x \cosh y - i \sin x \sinh y$, then show that $f''(z) = -f(z)$.

(g) Without evaluating the integral show that

$$\left| \int_C \frac{dz}{z^2 - 1} \right| \leq \frac{\pi}{3},$$

when C is the arc of the circle $|z| = 2$ from $z = 2$ to $z = 2i$ that lies in the first quadrant.

(h) Find the centre and radius of convergence of

$$\sum_{n=1}^{\infty} \frac{(3n)!}{2^n (n!)^3} z^n.$$

(i) Develop $\frac{1}{1-z}$ in negative powers of z and specify its region of convergence.

(j) Determine the location and type of singularity of $f(z) = \tan 2z$.

2. A string of length L is fastened at both ends points $x = 0$ and $x = L$ is initially in a position given by $y = y_0 \sin^3\left(\frac{\pi x}{L}\right)$. It is released from rest from this position, find out the equation of the subsequent motion. 10

3. A rod of length l with insulated sides is initially at a uniform temperature u. Its ends are suddenly cooled to 0 °C and are kept at that temperature. Find out the temperature distribution in the rod. 10

4. (a) State Green's theorem, and verify it for $F = (y^2 - 7y)i + (2xy + 2x)j$ where C is the unit circle. 5

   (b) Evaluate the line integral

   $$\int_C F \cdot r'(s)\, ds$$

   by Stokes theorem, where $F = [-5y, 4x, z]$ and C is the circle $x^2 + y^2 = 4$, $z = 1$. 5

5. (a) State and derive the Cauchy-Riemann equations in cartesian form. 5

(*b*) Show that the function $u(x, y) = 4xy - 3x + 2$ is harmonic and find its conjugate harmonic function. 5

6. (*a*) Find the value of the integral

$$\int_0^{1+i} (x - y + ix^2)\,dz$$

along the path $y = x^2$. 5

(*b*) Evaluate

$$\int_C \frac{4 - 3z}{z(z-1)(z-2)}\,dz,$$

where C is $|z| = 3/2$ using Cauchy integral formula. 5

7. (*a*) Find Taylor's expansion of

$$f(z) = \frac{1 + 2z^3}{z^2 + z}$$

about the point $z = 1$. 5

(*b*) Obtain the Laurent's series for the function

$$f(z) = \frac{1}{z(z+2)^3}$$

about $z = -2$. 5

8. Evaluate the following integrals by contour integration : 5 + 5

(*i*) $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2+1)(x^2+4)}$

(*ii*) $\int_0^{2\pi} \frac{d\theta}{3 + 2\cos\theta}$.
